

Activity 4: Newton's Law of Cooling

| Equipment | Quantity |
|----------------------------------|----------|
| Xplorer GLX | 1 |
| Fast-response Temperature Probes | 2 |
| Styrofoam Cups | 2 |
| Test Tube | 1 |
| Tape | 10 cm |
| Sand | 100 g |
| Hot Water (50 to 70 °C) | 500 mL |

Background

Newton's Law of Cooling states that the rate of temperature change of a body is proportional to its relative temperature, r , or the body's temperature minus the temperature of the surroundings. Mathematically, Newton's Law of Cooling can be written:

$$(eq. 1) \quad \frac{dT}{dt} = -kr$$

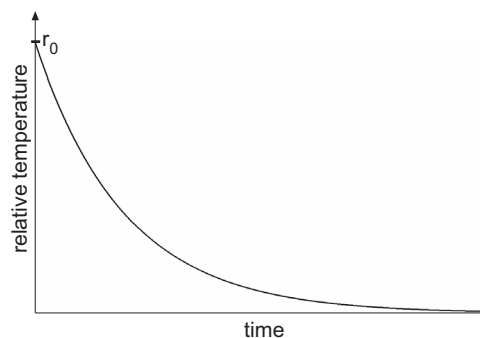
where dT/dt is the rate of temperature change and k is a constant greater than 0.

Another way to write Newton's Law of Cooling is

$$(eq. 2) \quad r = r_0 e^{-kt}$$


where r is the relative temperature at time t , and r_0 is the relative temperature at $t = 0$. A graph of Equation 2 is shown to the right.

In this experiment, you will record and graph the temperature of a hot object (a test tube full of sand) as it cools. You will then compare the experimental data with the theoretical graph.



Before You Begin

Start a new experiment on the GLX.

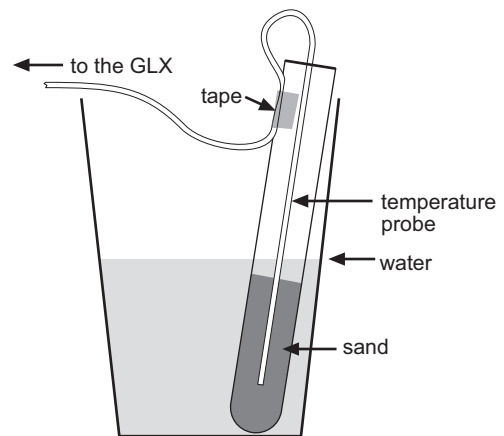
1. Press  to go to the Home Screen.

- Use the arrow keys to highlight the Data Files icon and press \checkmark to open the Data Files screen.
- Press $F4$ to open the Files menu and press 1_{page} to select New File.
- When the GLX asks if you would like to save the previous file, press $F1$ to save or $F2$ not to save.

Procedure

Equipment Set-Up

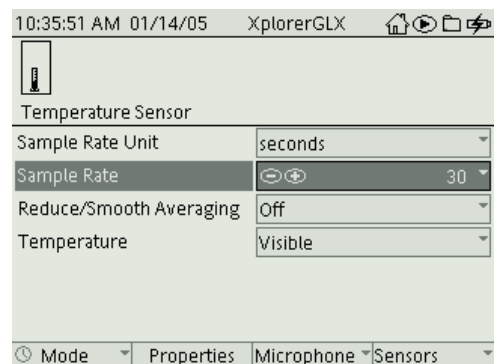
- Place the end of a fast-response temperature probe in the test tube, then fill the test tube about one-third full of sand, as pictured to the right. The probe should be embedded in the sand. Use tape to secure the probe.
- Partially fill a cup with hot water (50 to 70 °C) and immerse the sand-filled portion of the test tube. Let the sand warm up for a few minutes while you set up the GLX.
- Tape a second temperature probe to the inside wall of an empty cup. (This probe will be used to measure the temperature of the surrounding air.)



Immerse the sand-filled portion of the test tube in hot water

GLX Set-Up

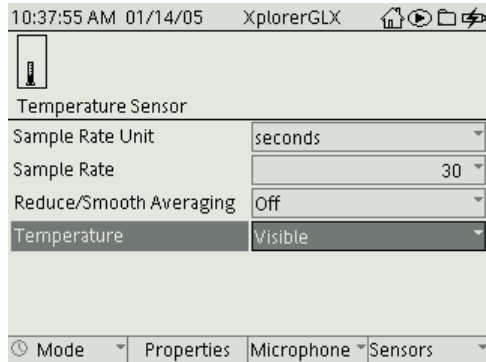
- Connect the first temperature probe to the GLX.**
 - Connect the temperature probe that is in the test tube to Temperature Port 1 on the left side of the GLX.
 - If there are other sensors connected to the GLX, remove them.
- Set the sampling rate to 30 seconds between samples.**
 - Press Home to go to the Home Screen.
 - Press $F4$ to open the Sensors screen.
 - Use the up and down arrow keys to highlight Sample Rate Unit. Press \checkmark , then press 2_{lux} to select "seconds."
 - Press the down arrow key to highlight Sample Rate. Press $+$ or $-$ to set the time between samples to 30 seconds.



Set the sampling rate to 30 s between samples

3. Rename the measurement.

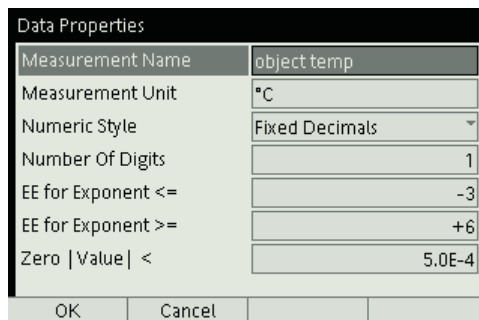
- a) In the Sensors screen, use the arrow keys to highlight the measurement named “Temperature”



F2

- b) Press **F2** to open the Data Properties box.
- c) Use the up and down arrow keys to highlight Measurement Name and press **✓** to make it editable.
- d) Type “object temp” and press **✓**.*
- e) Press **F1** to accept the changes and return to the Sensors screen.

*To enter text, use multipress text entry (see page 103) or an attached USB keyboard (see page 103).

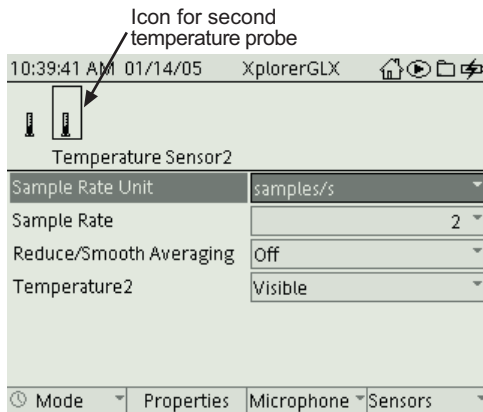


F1

F2

4. Connect the second sensor to the GLX.

Connect the temperature probe that will measure the air temperature to Temperature Port 2 of the GLX. You will see the new icon appear in the upper part of the Sensors screen with a box around it.

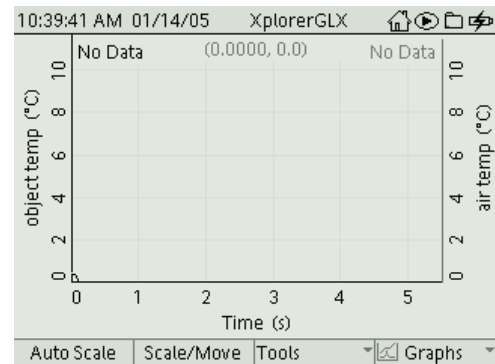


5. Set the sample rate of the second sensor and rename the measurement.

- Repeat step 2 to set the sampling rate of the second sensor to 30 seconds between samples.
- Repeat step 3 to set the Measurement Name to “air temp.”

6. Set up the Graph display to plot Object Temperature and Air Temperature versus Time.

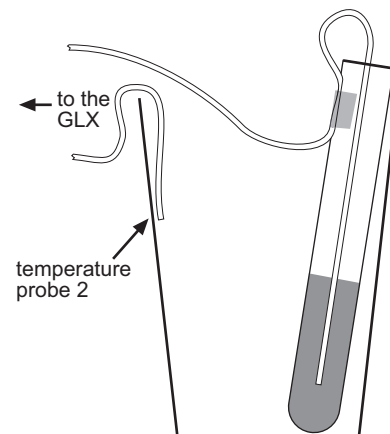
- Press Home to return to the Home Screen.
- Press $F1$ to open the Graph display. The display will be automatically set up to graph object temp versus Time.
- Press $F4$ to open the Graphs menu, and press $3way$ to select Two Measurements mode. Air temp will be added to the Graph.



Graph prepared to display object temp and air temp vs. Time

Data Collection

- Remove the test tube from the hot water and dry it thoroughly with a paper towel.
- Place the test tube in the empty cup so that the sand is several centimeters away from the air temperature probe.
- Press Play to start data collection.
- Press $F1$ to automatically scale the Graph.
- After the sand has cooled to about 30 °C, press Play to stop data collection.



Place the test tube in the empty cup so that the sand is several centimeters away from the air temperature probe

Analysis

Observe the graph of Object Temperature versus time and Air Temperature versus Time. If the experiment were allowed to run indefinitely, what would the relationship between object and air temperature eventually be?

1. Create a calculation for relative temperature.

- Press Home to return to the Home Screen; press $F3$ to open the Calculator.
- If you see the Num Lock symbol NL in the lower right corner of the screen, press $F4$ to open the Edit menu, then press 1para to turn Num Lock off.
- Enter:

$$r = [\text{object temp } (^{\circ}\text{C})] - [\text{air temp } (^{\circ}\text{C})] \quad \checkmark$$

Use multipress text entry to type "r".

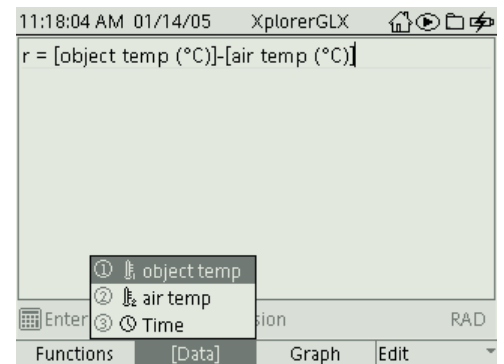
To insert [object temp ($^{\circ}\text{C}$)] and [air temp ($^{\circ}\text{C}$)] press $F2$ to open the [Data] menu, select the desired data from the menu, and select units of $^{\circ}\text{C}$.

- Remember to press \checkmark to complete the calculation.

2. Make a new graph of r versus t .

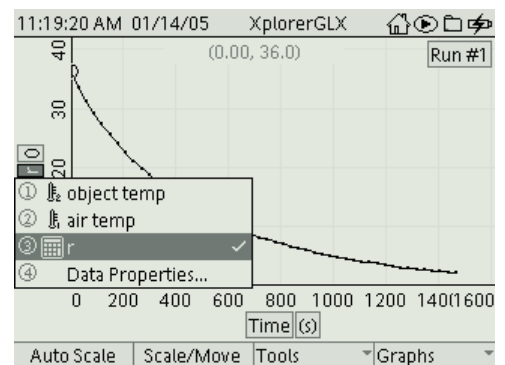
- Press Home to return to the Home Screen; press $F1$ to open the Graph.
- Press $F4$ to open the Graphs menu; press 6min to select New Graph Page.
- Press \checkmark twice to open the data source menu. Select r from the menu.

3. Does the graph of Relative Temperature versus Time appear to agree with Equation 2? _____



$F2$

Open the [Data] menu and select the desired data to insert into the calculation



4. What is the initial relative temperature at Time = 0?

To find the initial relative temperature, press the up arrow key to move the Data Cursor to the first data point.

Initial temperature, $r_0 =$ _____

In order to find the value of the constant k for this cooling curve, you can use another expression of Newton's Law of Cooling, derived from Equation 2:

(eq. 3)
$$\ln \frac{r}{r_0} = -kt$$

On a graph of $\ln(r/r_0)$ versus t , the slope will equal $-k$.

5. Create a calculation for $\ln(r/r_0)$

- a) Press $\left[\text{Home} \right]$ to return to the Home Screen; press $\left[F3 \right]$ to open the Calculator.
- b) On a blank line, enter:

 $\ln(r/r_0)$ $\left[\checkmark \right]$
- c) To insert the $\ln()$ function, press $\left[F1 \right]$ to open the Functions menu, use the arrow keys to highlight $\ln()$, and press $\left[\checkmark \right]$.

| | | | |
|-----------------|-----------------|------------------------------|-------|
| sin θ | cos θ | tan θ | ^ |
| arcsin θ | arccos θ | arctan θ | π |
| ^2 | log θ | lnθ | 0 |
| sqrt θ | 10^ | e^ |) |
| Functions | [Data] | Graph | Edit |

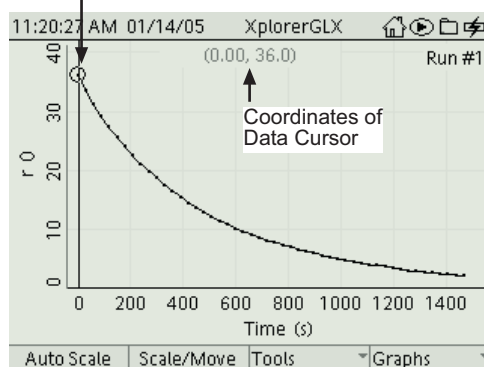
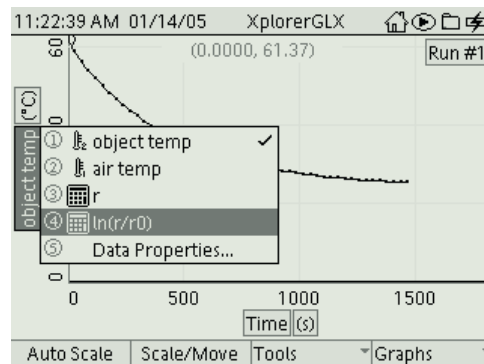
$\left[F1 \right]$

- d) The Calculator will prompt you on the next line to enter the value of r_0 . Type the initial relative temperature and press $\left[\checkmark \right]$.

6. Make a graph of $\ln(r/r_0)$ versus t .

- a) Press $\left[\text{Home} \right]$ to return to the Home Screen; press $\left[F1 \right]$ to open the Graph display.
- b) Press $\left[F4 \right]$ to open the Graphs menu; press $\left[6^{\text{mno}} \right]$ to select New Graph Page.
- c) Press $\left[\checkmark \right]$ twice to highlight the data source menu. From the menu select $\ln(t/r_0)$.

Press the up arrow to move the Data Cursor to the first data point.

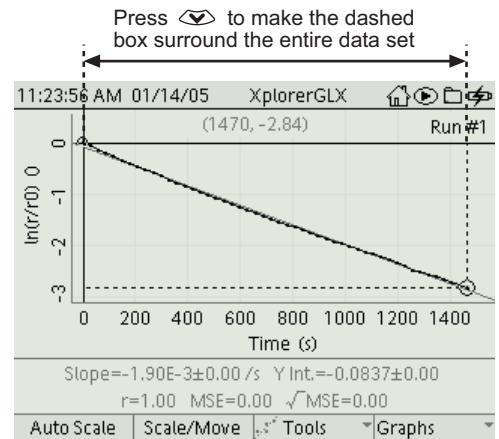



7. Apply a linear fit and find the value of k .

- Press $F3$ to open the Tools menu; press 5_{IN} to select Linear Fit.
- Press the down arrow key to make the dashed box surround the entire data set.
- The slope of the best-fit line equals k .

$k =$ _____ (include units)

Now that you have experimental values for the constants r_0 and k , you can substitute them into Equation 2 and compare that model to the collected data.

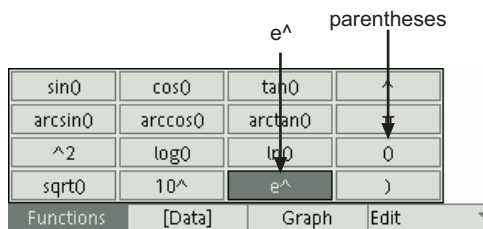


8. Enter Equation 2 into the Calculator using your experimental values of r_0 and k .

- Press \leftarrow to return to the Home Screen; press $F3$ to open the Calculator.
- On a blank line, enter:

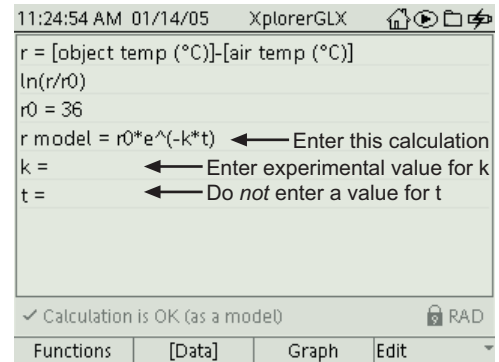
$r \text{ model} = r_0 * e^{(-k*t)}$ ✓

- To insert the function e^x , press $F1$ to open the Functions menu, use the arrow keys to highlight e^x , and press ✓.



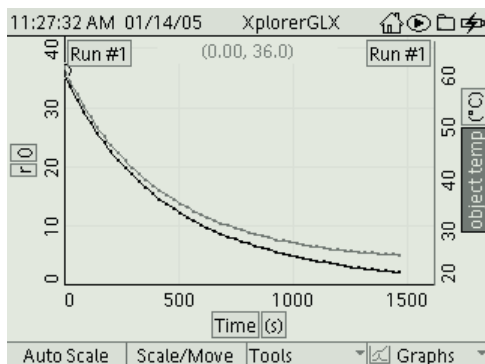
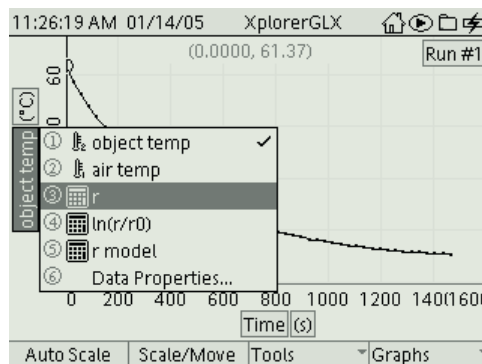
Also open the Functions menu to select and insert the pair of parentheses.

- The Calculator will prompt you to enter the values of k and t . (It does not prompt you for r_0 because you entered that constant in step 5.)
- Enter the value of k that you found in step 7 and press ✓.
- Time is the variable that will be plotted on the horizontal axis, so do not enter a value for t .

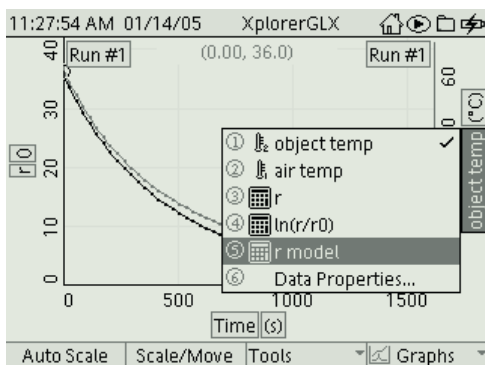


9. Compare collected relative temperature data with the modeled data.

- Press $\left[\text{Home} \right]$ to return to the Home Screen; press $\left[F1 \right]$ to open the Graph display.
- Press $\left[F4 \right]$ to open the Graphs menu; press $\left[6_{\text{mode}} \right]$ to select New Graph Page.
- Press $\left[\checkmark \right]$ twice to highlight the data source menu. From the menu, select r (the collected data).
- Press $\left[F4 \right]$ to open the Graphs menu; press $\left[3_{\text{mode}} \right]$ to select Two Measurements mode.
- Press $\left[\checkmark \right]$ to turn on the highlight, then press the left arrow key repeatedly until the second data source (on the right side of the Graph) is highlighted.



- Press $\left[\checkmark \right]$ to open the data source menu and select r model.



10. Does the modeled equation agree with the collected data? If it deviates, can you explain why?
