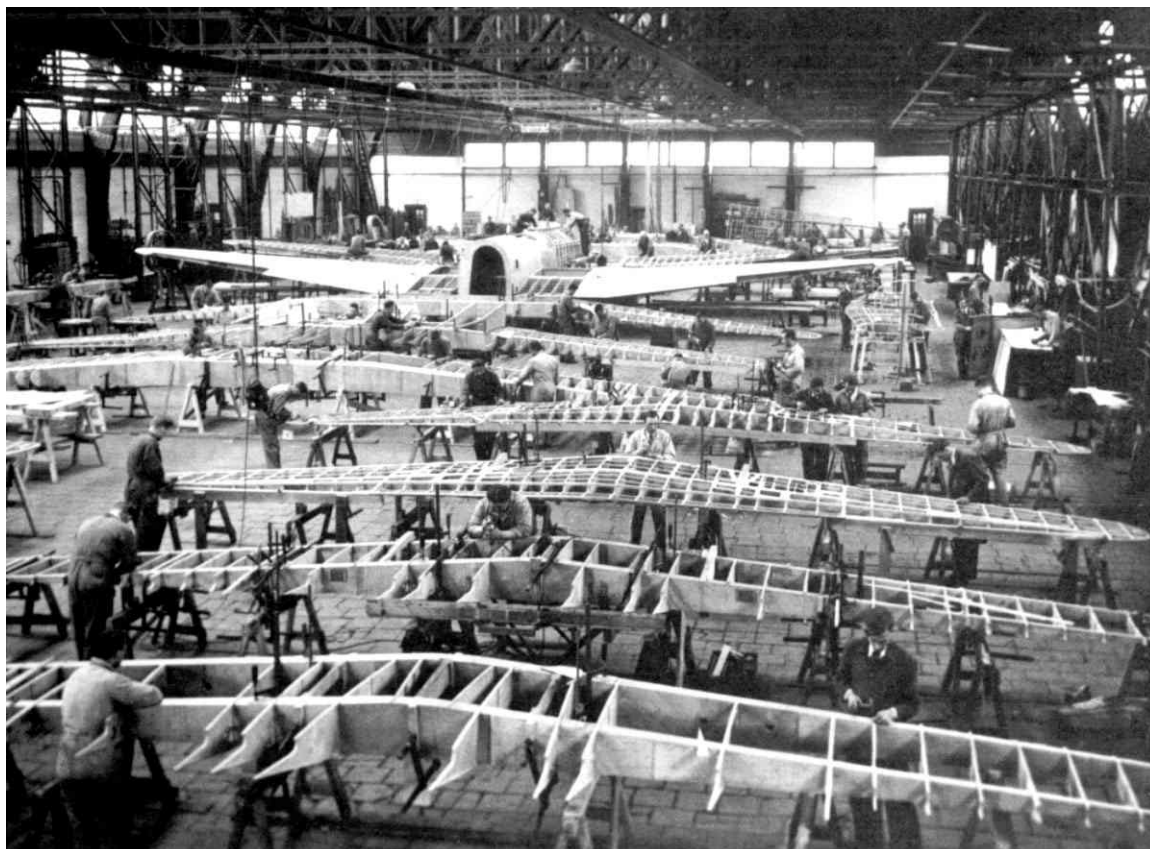


WORKING WITH BREAKS



Preliminary round for the 19th Mathematics A-lympiad

23 November 2007



Procedure for the preliminary round assignment for the 2007/2008 Mathematics A-lympiad

This Mathematics A-lympiad assignment contains one introductory assignment, two follow-up assignments and a final one.

General advice for working on this assignment

- First read the complete text of the assignment, so that you know everything you have to do.
- **Keep an eye on the time you spend on the introductory and follow-up assignments, be sure to have enough time left for the final assignment.** Divide tasks where possible and discuss when needed.
- If you have divided tasks within your group, discuss the results of the previous assignments with each other, before you start working on the final assignment.
- In the final assignment it is important that you explain the two alternatives as clearly as possible and give an analysis of the advantages and disadvantages.
- The answers of the three introductory assignments do not belong in the elaboration of the final assignment. Add the results of the introductory and follow-up assignments in an appendix. Remember the worksheets containing the graphs!

To be handed in:

- A complete description of two possible daily schedules, including an analysis of the advantages and disadvantages.
- As an appendix: the results of the introductory and follow-up assignments.

The jury will receive copies of your work; these copies must of course be legible.

Therefore, use a black pen to write, only print on A4 paper, do not make drawings with a pencil. If in any doubt, make a test copy!

Judging

Among other things, the jury will pay attention to:

- legibility and clarity of the final assignment,
- whether the work is complete,
- the use of mathematics,
- the arguments used and justification of choices (realism may play a part here),
- the level to which the assignment has been done,
- presentation: form, legibility, structure, use and function of appendices, etc.

good luck and have fun

We dedicate the 19th edition of the Alympiad to Berend Wielens.
Berend has set the tone for this assignment, but he never heard the music.

Introduction

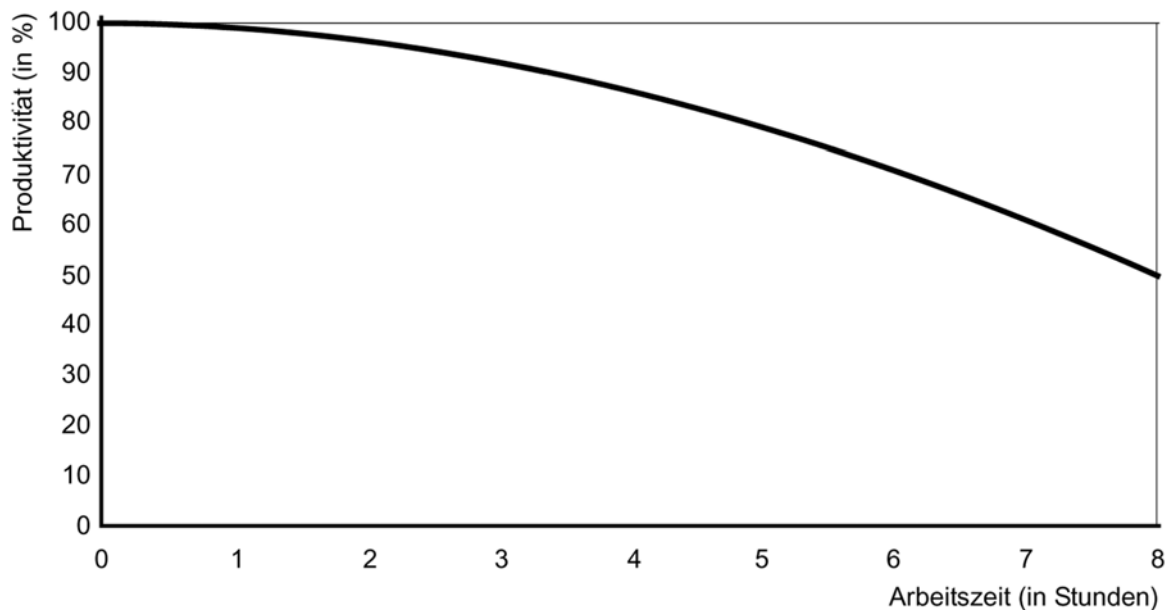
Everybody has experienced that it's not possible to just keep on going at work. Leaving aside that you will, need to eat for example, most work will physically tire you and after some time your concentration will become less. A truck driver who goes on without a break for ten hours, a teacher who does eight hours of correction without stopping, or a journalist who sits at his computer continuously typing for nine hours ... fatigue and loss of concentration can cause big or small disasters in all these cases.

Besides, with most work there will be a loss of productivity. This can be measured most easily with production work in a factory.

So, taking a break is necessary, but... what is the best way to divide those breaks? A few breaks, but longer ones, or a lot of short breaks, or a mix of the two: how to make the optimal schedule.

Introductory assignment

From experience and a study in a large German production firm the following global picture has emerged on the connection between productivity and the number of hours worked. (A more detailed graph can be found in Appendix 1)



The more hours you work, the lower your productivity will get. In the graph you can see, among other things, that after eight hours of working non-stop, so without any breaks, your productivity will have fallen to 50%.

From other studies we know that taking a break raises productivity. Just after a break, a worker's productivity is higher than just before one. Or to put it another way: productivity is back at an earlier, higher level. The study has resulted in the following rules of thumb:

- After a break within the first five hours of working (that is pure working hours) productivity will be back at the level of the time that is 3.5 times the length of the break before the start of that break.

An example helps to clarify the rule of thumb: if a worker starts at 08:00 and works until 11:25, by 11:25 his productivity has dropped to 90% of his maximum.

If the worker then takes a 40 minute break, by the end of that break he'll resume his work at a productivity level of $3,5 \times 40 = 140$ minutes before 11:25. Check, using the graph, what that new productivity level is.

- After a break that is taken after more than five working hours the effect is a bit smaller: in that case productivity after a break will be back at the level of three times the length of the break before the break.

Assignment 1

In the company that was mentioned before, the working day starts at 8 in the morning and ends at 5 in the afternoon. At 12:00 there is a lunch break, which lasts an hour. So the working day lasts nine hours, eight of which are actual working hours.

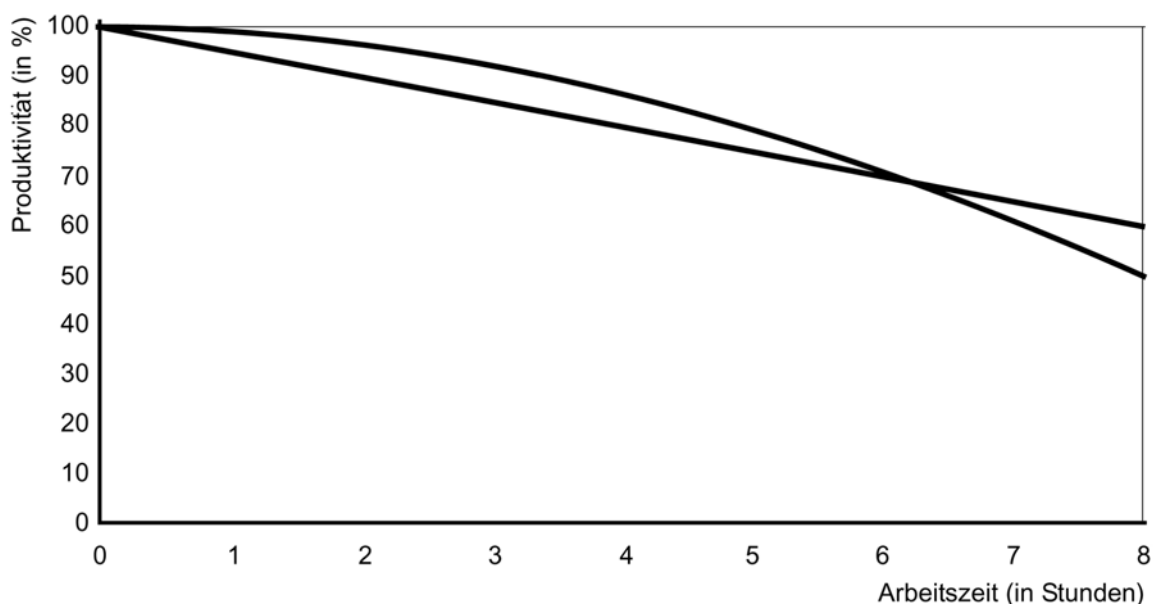
A worker produces a maximum of 600 wpu (work production-units) per hour. This 600 wpu is the *maximum productivity*.

The company board of directors is mainly interested in workers' total productivity over the whole working day.

- Make an estimate, based on the graph of the total productivity for eight hours of continuous work. Use the graph in Appendix 1 for this.
- Also estimate the production for the given daily schedule, with a one hour break between 12:00 and 13:00, and work between 08:00 and 17:00. Clearly indicate how you have used the graph.

Follow-up assignments

To make it easier to do the productivity calculations, a decision is made to simplify the model to a fitting linear model. You can find the graph for the adjusted model below, together with the graph for the original model:



As you can see, the new model assumes that, based on 8 hours of continuous work without a break, productivity will fall in a straight line from 100% to 60%.

Assignment 2

- a Calculate total production per worker for eight hours of continuous work. Use the worksheets in Appendix 2
- b Also calculate a worker's production for the original daily schedule (the one with working hours between 08:00 and 12:00 and between 13:00 and 17:00). Do not just give your calculations, but also explain using the graph on the worksheet in Appendix 2.
- c Find out if productivity rises if you split the 1 hour break in a number of shorter breaks (which must all be the same length) with a total length of an hour. Where would you plan those breaks and what will be the maximum obtainable production in that case?

Assignment 3

Most workers prefer to have as much (continuous) free time as possible. So for instance an extra (half) day off, or all working days shorter. If that isn't possible, most workers would prefer long breaks.

The company's board of directors agrees with all possible work- and presence scenarios, provided that a worker can organise his schedule so that he (or she) can bring in a productivity of at least 19.200 wpu per week. The factory is open every day between 07:30 and 18:30.

- Find out if this is possible for someone who wants to work four days.
- Find out attractive options for workers who stay on a five day week.

Give the accompanying daily schedules for all options and represent them graphically in one of the worksheets in Appendix 3.

Final assignment

Of course it's not only the employer who determines rules for working hours. There are also health and safety (ARBO) rules that imposes all kinds of limitations on the daily schedule. These rules of course also help to protect the workers! On the next page you can find some of the health and safety rules that apply to the company.

The board of directors wants the highest possible production; the worker wants as much free time as possible.

Give at least two well-founded proposals for a daily schedule for the workers, that the works council and the board of directors can together make a choice from. Take into account:

- the interest of both employer and employee (worker)
- health and safety rules and
- the minimum of 19.200 wpu per week.

List the working hours and the daily schedule and determine the level of production that can be achieved with them. In any case, use graphics to support your proposal.

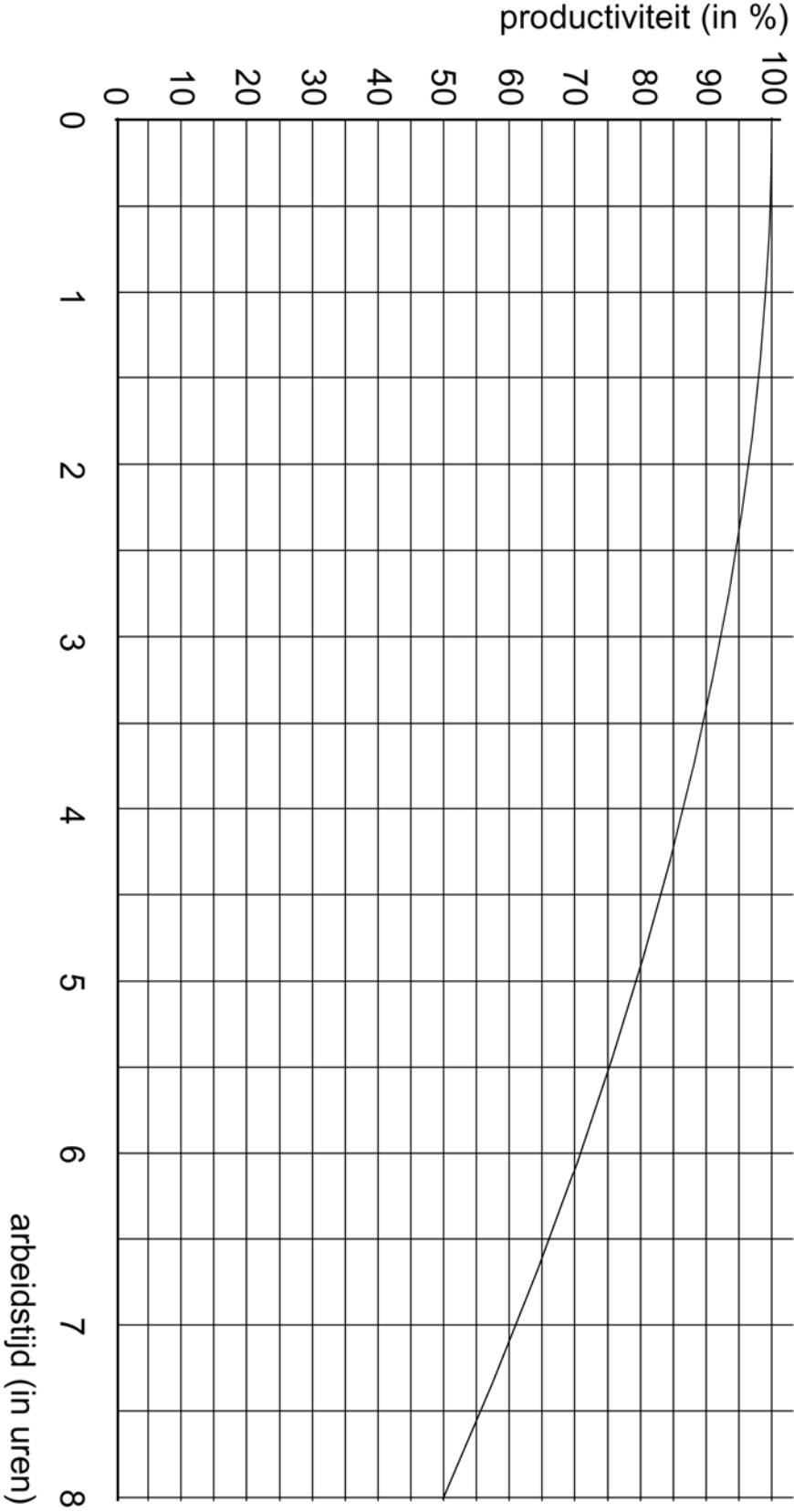
Also mention all considerations that have been taken into account, the advantages and disadvantages, and take care to give a clear motivation for the criteria used!

Health & Safety rules

Breaks: *If you have a working day of:*

- *more than five and a half hours, you have at least 30 minutes of continuous rest break;*
- *more than eight hours, the break time will be at least 45 minutes, 30 of which will be continuous;*
- *more than ten hours, the break time will be at least 60 minutes, 30 of which will be continuous.*

Appendix 1
[productivity]/[working hours]



Appendix 2
[productivity]/[working hours]

