

CROSS CURRICULAR TEACHING IN THE SUBJECTS OF MATHEMATICS AND LANGUAGE (DANISH/ENGLISH)

Bjørn Felsager

Midtsjællandske Gymnasier, Haslev afdeling, Denmark

In Denmark a recent curriculum reform has introduced a new cross curricular subject where traditional subjects combine to investigate a particular theme that benefits from methods used in the participating subjects. At my own high school we have developed a module between Mathematics and Danish involving argumentation and logic, in which we investigate in particular logical puzzle solving using computers. In this paper I discuss the contents of the whole module in general and how to formalize puzzles so that they can be solved automatically by computer in particular.

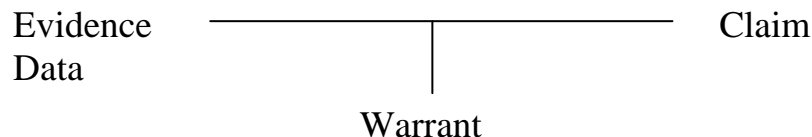
ON THE CONSTRUCTION OF A MODULE DEALING WITH ARGUMENT AND LOGIC IN THE DANISH UPPER SECONDARY SCHOOL

In Denmark a recent curriculum reform from 2005 has emphasized the importance of cross curricular teaching and has furthermore introduced a new cross curricular subject under the heading "General study preparations" (AT), where several subjects combine to investigate a particular theme that benefits from methods used in several subjects. Special blocks of lessons lasting from 3 to 5 days are allocated to AT. The students attending the General upper secondary program in high school, STX, are divided into different study programmes combining three subjects from the humanistic, social or natural sciences (which includes mathematics). Further details about mathematics in the new reform can be found in the recent international evaluation report: The subject of Mathematics from an international perspective (2009) commissioned by the Danish Ministry of Education.

In 2008 the association of Danish Science Gymnasiums, DASG (2006), in collaboration with The National Centre for Information about the Didactics of Mathematics, NAVIMAT (2008), decided to establish a pilot programme developing new cross curricular materials for teaching mathematics. At my high school we joined this programme. The General study preparations, AT, favour cross curricular teachings with subjects from a different faculty, such as the humanistic or social sciences. We therefore decided to focus on the cross curricular teaching with the humanistic sciences. We already had some experience with combining Mathematics and History and therefore opted to focus on cross curricular teaching with a language subject, in our case Danish which is mandatory for all STX students. In the preceding school year 2008-09 we therefore set up two experimental classes 1c and 1d which deliberately were chosen not to come from the Science study programmes, and so they had only mandatory mathematics at the lowest level MAT C in their first year of STX.

Toulmin's argumentation model

The group of Math- and Danish teachers decided to work with *logic and arguments* as the common theme. First we had to choose a common framework to emphasize the unity of argumentation: We opted for the argumentation model developed by the British philosopher Stephen Toulmin (1984), since this is in wide spread use by the humanistic and social sciences and we believe that it can easily be adapted to mathematics as well. In Toulmin's restricted model you want to *justify* a **claim** by some **evidence** or **data** and a **warrant** that links the observation to the claim:



In mathematics the **claim** corresponds to a logical statement/theorem Q that we want to prove, the **evidence** is another statement/theorem P , that we have already proved from which Q is supposed to follow, and the **warrant** is the theorem, that P implies Q (i.e. $P \Rightarrow Q$), which is yet another theorem, that must be shown separately.

Although traditionally mathematical reasoning is more concerned with deduction than justification, in the actual process of doing experimental mathematics you typically use some kind of induction to put forward some hypothesis, that you subsequently want to justify. So Toulmin's method of reasoning apply very well to mathematics despite the fact that general schemes like Toulmin's model are rarely mentioned in standard high school text books for mathematics. In the professional literature the use of Toulmin's model in mathematics is discussed in e.g. Aberdein (2006).

Preparations for the module on argumentation and logic

To get acquainted with Toulmin's model as well as the art of solving logical puzzles we agreed to initiate the project with two math lessons (90 minutes) covering on the one hand Toulmin's model of argument in mathematics and on the other hand introducing logical connectives and the idea of using truth tables to solve logical problems. Since we were dealing with a humanistic study programme we decided to introduce Toulmin's argument model using elementary number theory as the playground: Even/odd numbers, square numbers etc. Students should come up with definitions, various hypotheses concerning parity rules etc. In the section with logical puzzles we used love puzzles from the American logician Raymond Smullyan (1978) "What is the name of this book" to illustrate the general ideas behind the use of logical connectives in particular the imply command: If..., then ..., as well as truth tables for compound statements as can be seen in the following example:

Suppose the following two statements are true:

- (1) I love Betty or I love Jane.
- (2) If I love Betty then I love Jane.

Does it necessarily follow that I love Betty? Does it necessarily follow that I love Jane?

Outline of the module structure

The module was allocated 4 days covering 14 lessons (90 minutes). It was initiated with the Shakespeare movie: *The Merchant of Venice*, directed by Radford (2004) which combines a love story with the famous story about the merchant who loses his fortune and therefore are bound by his oath to pay the Jewish moneylender with his own flesh. This gives rise to dramatic dialogues in the court room that can be analyzed using Toulmin's model. In the love story the fair Portia is courted by several suitors, but to win her they must pass a special trial set up by her deceased father: They have to choose among three caskets the casket containing her portrait. To help the suitors choose the right one, the caskets carry statements such as the gold casket: 'Who chooseth me shall gain what many men desire;' but like oracle statements they are ambiguous, e.g. we are not told what it is, that many men desire: Gold or women.

In the next 4 modules they were introduced to rhetoric (by a teacher in drama), argument analysis (Danish) and puzzle solving (Mathematics). Subsequently they should work in small groups, where each group had to (1) prepare their own puzzle which should be solvable by reasoning as well as by computer using truth tables, (2) prepare their own debate (e.g. in the form of a debate program on TV) using one of the following three themes based upon the movie: "For every one there is a special one, the one and only!", "Everyone is equal!" and "You must never defy your parents' will!". On the last day the students should present their puzzles and their debate in front of the class, which should then try to solve the puzzle. Marks were given on the basis of this presentation as well as on a written summary prepared by the groups.

Using computer to solve logical puzzles

The idea of using truth tables to solve logical puzzles is of course not new, neither is the idea of using a computer to automatize the handling of truth tables. I first learned the techniques from the inspiring notes by Dean Clark (1994). The transformation of Portia's puzzle from an oracle type to an logical type of puzzle is due to Smullyan (1978) in his classic text "What is the name of this book". In Smullyan's version the statements on the three caskets are given as follows:

Gold casket	Silver casket	Lead casket
The portrait is in this casket	The portrait is not in this casket	The portrait is not in the gold casket

Portia explained to the suitor that of the three statements, at most one was true. Which casket should the suitor choose?

The ambiguity in the logical puzzle is now related to the fact, that we do not know if a particular statement is true or false, we only know that at most one is true! To solve it by computer we use the list and spreadsheet from TI-Nspire (as part of the DASG project all the humanistic study programmes this year used TI-Nspire CAS).

First we introduce three possible universes corresponding to the three possible locations of the portrait, as well as the content variables **portrait** and **blank**, which allows us to make statements about the placement of the portrait. Finally we translate the general rule that at most one statement is true into a formal logical statement:

	A	B	C	D	E	F	G	H	I	J	K	L
	portrait	blank	gold	silver	lead	=gold=portrait	=silver=blank	=gold=blank	=(not (gold			
1	portrait	blank	portrait	blank	blank	true	true	false	false			
2	portrait	blank	blank	portrait	blank	false	false	true	true			
3	portrait	blank	blank	blank	portrait	false	true	true	false			
4												
I	rule:=(not (goldstatement and silverstatement or silverstatement and leadstatement or leadstatement and goldstatement))											

Evaluation

The module was evaluated by a standard questionnaire we use for evaluating AT-modules. It was very well received by the students: Out of 37 humanistic students 34 recommended a repetition for the next classes. Similarly 15 students found a very good coherence between Mathematics and Danish in the module, 21 students found a good coherence and only one student found a bad coherence. This also reflects the impression from the teachers involved. The module thus addresses one of the main problems according to the international evaluation of Mathematics after the reform: Lack of good examples of cross curricular teachings, that benefit from both subjects.

References

- The subject of Mathematics from an international perspective* (2009). Available from <http://www.eva.dk/projekter/2008/fagevalueringer-paa-de-gymnasiale-uddannelser-2008/rapporter/the-subject-of-mathematics-from-an-international-perspective>.
- DASG (2006). *Danish Science Gymnasiums*. Website: <http://www.emu.dk/gym/tvaers/sciencegym/english.html>
- NAVIMAT (2008). *The National Centre for the information about the Didactics of Mathematics*. Website: <http://www.navimat.dk> (no translation available may 2009).
- Toulmin, S., Rieke, R. D. & Janik, A. (1984). *An Introduction to Reasoning*. New York: Macmillan
- Aberdein, A. (2006) *The uses of Argument in Mathematics*. Ch. 22 in Hitchcock, D. & Verheij, B. (Eds.) *Arguing on the Toulmin Model, New essays in Argument Analysis and Evaluation*. Springer.
- Smullyan, R. (1978). *What is the name of this book*. Prentice Hall. The love puzzle is discussed in chapter 8: *Logical puzzles*. Portia's puzzle is discussed in chapter 5: *The mystery of Portia's casket*.
- Clark, D. (1994). *Mathematics of the unexpected, Symbolic Mathematics on the Macintosh*. Courseware for MTH 108. The University of Rhode Island. Clark who is inspired by Smullyan discusses puzzle solving using the CAS-program Theorist in the last three sections: *The logical functions, The bizarre planet of saints and sinners and Asylum ... on the planet of saints and sinners*.